Method of Genetic Algorithms for the Optimal Investment Portfolio

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***Abstract*—** this paper is devoted to the problem of optimal investment portfolio design on the base of mathematical modeling tools and method of genetic algorithms. The purpose relates to investing the funds into financial assets such that certain requirements regarding the expected profits and possible losses would be reached. The main result is developing a state-space dynamical model of portfolio management and applying a genetic algorithm in order to obtain the optimal solution.

I. Introduction

The optimization problem of an investment portfolio is one of the main problem in financial management and engineering so it represents both scientific and practical interest. The effectiveness of investments essentially depends on chosen portfolio management strategies. The development and research of such strategies requires the use of modern mathematical methods, models, computing technologies. This is a well-known problem with many different solution's ideas and methods [1,2,3]. In the work [4] authors propose one of the approaches for a rational choice of an investment portfolio based on the criterion of mean and standard deviation. Article [5, 6] are devoted to the investment portfolio problems of the industrial enterprise, calculation of profitability and risk. The question of optimizing the investment portfolio on the basis of mathematical modeling is also presented in [7-10]. The semi-Mark's control processes for investigating the multi-component model of the theory of inventory management was considered in [11]. The task of making investment decisions under uncertainty is presented in [12], and in [13] an approach to model the asset prices is proposed, which allows obtaining high simulation and forecast properties of the returns of these assets. The work [14] investigates multi-stage stochastic portfolio optimization problems with the use of risk measures. Since the stock market serves as an effective mechanism of investments attraction, the effectiveness of investments crucially depends on the chosen strategies of investment portfolio management.

In this paper a one-period problem of portfolio management is considered, which is related to the problem of optimal investment decisions without any possibility of changing a portfolio composition during the investment period. One of the most famous optimization methods for one-period portfolios is the model of the mid-dispersion constraint, proposed by Markowitz [15,16], where the problem is solved by quadratic programming and related type of the models are discussed. In these works the connection between the problem of long-term portfolio management and control engineering is explained. In the same time a discussion why stochastic optimal control is the appropriate framework for solving the problem of multi-period portfolio optimization is given. In context of this problem Markowitz proposes the most famous and most popular model that is the mean-variance framework. This model defines the expected returns and the risk as the variance of a portfolio. The problem can be efficiently solved by a quadratic programming. A lot of interesting results relating of the models of single-period portfolio are: the mean deviation approach of Konno and Yamazaki [17], the regret optimization approach of Dembo and King [18], the min-max approach described in [19], as well as the conditional value-at-risk approach of Rockafellar and Uryasev [20]. They mostly differ in the methods of modeling of return distributions and risk measures.

In this article we search an effective management strategies of investment portfolio. We consider portfolios of financial assets such as stocks, bonds, indices of various markets, and cash. A portfolio is described by two main characteristics: return (expected profit) and risk (possible losses). Very often, the risk aversion defines in which assets investors tend to invest, portfolios with high expected returns are in general more risky. But the risk return pattern is not the only variable that defines a portfolio. Other important variables are the investment horizon (duration of the investments), the asset classes, or the markets (domestic or international). In this paper we develop the dynamical model of portfolio management for long-term investment. The basic idea is to use method of dynamical analogies [21-23] in order to construct the state-space model. Description of the dynamical processes is based on a system of differential equations. Using this model we solve the multi-criteria investment portfolio optimization problem using cluster genetic algorithm based on the method presented in [24]. The results were tested on developed an appropriate software.

In the Introduction Section we discuss the base problems of the investment portfolio investigation, we analyze the existing approaches to model it and review the related economic problems; The Section II is devoted to development of the

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model of forecasting the price of risk-free assets, the appropriate theoretical base of the dynamic modeling is presented. Based on the obtained model we develop an optimization algorithm in order to maximize a profit of the investor in the Section III. In the Section IV the software is presented for finding the optimal investment portfolio on the base on the proposed algorithm.

II. MODEL OF FORECASTING THE PRICE OF RISK-FREE ASSETS

In this section we explain the dynamical effects occur in the process of investment portfolio management. Consider the problem of dynamic portfolio management of risk-free assets and the optimal market of capital.

The basis of the model construction is the method of dynamic analogies, which has been widely used since the time of Walras [21]. This method which applies the laws of dynamics in economic problems is described in [22-23]. We use an analogy with the laws of dynamics regarding the specially chosen measure of motion to some extent. As such property we define a quantitative measure of the price dynamic of assets of the investment portfolio $x(t)$. As the speed of prices changing we enter the value $\dot{x}(t)$ and define it as the *price change rate*.

The result of modeling is presented in a form of a system of ordinary differential equations, which describes the change of the prices of the assets depending of time and satisfies a number of assumptions - hypotheses concerning the economic relations between the parts of the system.

Suppose we have the functions of prices of assets are determined by available measurable values $x_1(t), x_2(t),..., x_n(t)$ at the certain time t. Based on the considerations similar to [22] when constructing a model, we make the *following assumptions.*

1) The dynamics of prices are defined by economic forces that affect them and change. According to the planning process of investment activity, we describe the nature and character of these active forces that are generated by the subjects of the process. These forces are divided into two groups that they affect on the prices by contributing to growth and decline, respectively. They are due to the need to spend or to receive revenues. Based on the laws of dynamics in relation to a specially chosen measure of motion (changing), we define these economic forces as follows:

$$F_i(t) = C_i(t) - P_i(t),\ i = 1,...,n \qquad (4)$$

The economic forces $C_i(t)$ are the forces that influence the process in the direction of decreasing the price of assets (the results of operating, investment, financial activities: taxes, fees, production costs of sales, sales costs, production management costs, costs for technology improvement and organization process of production).

$P_i(t)$ includes forces that reflect the impact on the dynamics of asset prices in the direction of their growth, for example: financial receipts, contributions of founders into working capital, reserve capital, capital formed by the bank's long-term loan.

2) The principle of the independent action of economic forces relating to change prices [23] is hold. At the same time, the rate of prices changing under the influence of forces is equivalent to the total effect of all "applied" economic forces. All economic forces operating on this process are in equilibrium.

3) The following law is used: the rate of increase (decrease) of the price of asset is proportional to the resulting sum of all operating economic forces. Each economic force has the physical dimension of the rate of growth, that is, the rate of price changing per unit time.

In accordance with assumption 2 (the principle of independence of the actions of economic forces), the total effect of forces that influence to the dynamics of the process, is expressed by the following sum of functions:

$$F(t) = \sum_{i=1}^{n} F_i(t) \qquad (5),$$

where n – is amount of applied economic forces.

Therefore, we use $\dot{x}(t)$ - the rate of price change. Then we construct the state-space mathematical model for the price dynamic in the form of a ordinary differential equations. Thus, the problem is defined as the price forecast of risk-free assets, namely as the dynamic of capital market.

First, we define the dynamic equations of risk-free assets:

$$\dot{x}(t) = r(t)\,x(t) + F(t), \qquad (8)$$

where $x = \{x_s(t)\}$ – state vector of the system, that describes the price dynamic of i -th asset, $r(t)$ – rate of return, $F(t)$ – sum of economic forces that influence to the risk-free asset prices.

Suppose an investor has the opportunity to form a portfolio of *N* risk-free assets at the initial time t_0. Although the assumption of risk-free condition restricts the applicability of the model, note that these arguments are valid in some cases. For example, the price of forward contracts are usually calculated as the risk-free asset prices.

Forward contract is a standard document certifying individual obligation to purchase (sell) an underlying asset at a certain time under certain conditions in the future, with fixed selling prices when concluding the contract.

Functions of risk-free assets yield the dynamic equation

$$\dot{r}(t) = kr(t) + F_r(t). \qquad (9)$$

Identification of the parameter k in this equation is possible by assumption that the value of this function is known in the specified period of t_0 up to time $t > t_0$. This assumption is based on the fact that although the real market system has a complete or partial uncertainty of external influences, but the information about yields can be obtained, by knowing the price of forward contracts of related assets maturing at some time t in the future.

$F_r(t)$ is a sum of economic forces affecting the dynamics of return. The economic forces $F(t)$ та $F_r(t)$ depends of the instability of exchange rate; the refinancing rate of the central bank; state support of certain enterprises in the form of loans and benefits. They affect to the dynamics of their risk-free assets and rate of return.

The developed model of the optimal investment portfolio is based also on the approaches given in the paper [15], which presents a model for dynamics of investment asset prices using of stochastic differential equations. Basis of this model is the description of the dynamics of asset prices, which depend on changes in economic and financial factors that describe the state of the economy. At the same time, the expected profitability of asset price dynamics is the function of these factors. Using also considerations and the principle given in [30], we find an optimal investment portfolio taking into account the dynamics of yield and rates of returns. In this case, the available funds U can be used to acquire n kinds of securities in amounts $U_1,...U_n$. The initial value α_i of one unit of type i is known, as well as its estimated value β_i at time t. It is assumed that $\beta_i > \alpha_i \quad (i = 1,...,n)$. The declared problem is to determine the composition of securities and assets such that the profits on their sales at time t would be maximum.

Therefore we propose an modification of this mathematical model, in order to make it more flexible for the potential user of investment portfolio. For this optimization we enter x_i - amounts of each of the assets as variables in the mathematical model. In addition to improve "practical potential" of tasks we associate an assessment of return with each asset (9) calculated on the basis of stock market securities, instead of projected income (8).

Thus, taking into account of (8)-(9), the resulting model of forecasting of total price of risk-free assets would be:

$$\begin{cases} \dot{x}(t) = r(t)x(t) + F(t) \\ \dot{r}(t) = kr(t) + F_r(t) \\ f = \max \sum_{i=1}^{n} \gamma_i \frac{\alpha_i x_i}{\sum_{i=1}^{n} \alpha_i x_i} \\ \sum_{i=1}^{n} \alpha_i x_i \le U \\ \gamma_i = \overline{r_i}, r \in [t_i, t_{i+1}], i = 1 \ldots n \end{cases} \quad (10)$$

where γ_i - expected return of i-th asset, $\frac{\alpha_i x_i}{\sum_{i=1}^{n} \alpha_i x_i}$ - the rate of the price of i-th asset, $x_i \in [x_i^-, x_i^+]$ - lower and upper limit of the amount of securities of type i, its specification is given in the Section III.

III. OPTIMIZATION ALGORITHM FOR THE INVESTMENT PORTFOLIO

A. Basic concepts

In this section we develop an algorithm in order to maximize a profit of investor which has the financial means of amount U, used to purchase n types of securities in the amount $U_1,...U_n$. Namely: to determine a composition of securities such that a profit on their sales at certain time t would be maximal. In order to solve this problem we use a cluster genetic algorithm. The algorithm was tested on developed software environment.

One of the variants of this optimization problem's solution is based on the modification of the classical genetic algorithm (GA) in the cluster GA [24]. Genetic algorithms - is adaptive search, which recently used to solve optimization problems. They are used as an analogue mechanisms of genetic inheritance and an analogue of natural selection. Cluster modification of genetic algorithm partially imitates principles of diversity in the population and the process of genetic selection. Namely, the partition of search space into sections depending on the value of the objective function.

Define the basic specifications of this problem.

In terms of GA, the number of securities of one company is a gene (chromosome), and a certain set of genes - is one of the realization of an investment portfolio. To determine the optimality of the chromosome, a fitness function was defined that takes into account the specifics of the solvable problem: obtaining the maximum return of the investment portfolio, the investor's desire to invest the whole amount of fund in the portfolio or obtain some balance for the formation of the portfolio, as well as strict control of the amount of allocated funds. As a result, the target fitness function of adaptability has the form:

$$f = \max \sum_{i=1}^{n} \gamma_i \frac{\alpha_i x_i}{\sum_{i=1}^{n} \alpha_i x_i} \quad . (11)$$

The basic terms of the declared problem are following.

Chromosome - a binary string consisting of zeros and units obtained as a result of the binary coding of the data set corresponding to the variant of the investment portfolio.

Population - a set of chromosomes, i.e. a set of implemented variants of the investment portfolio.

Adaptability - is defining a function to be optimized.

Crossover is an exchange of chromosomes by its parts to create a new population, i.e. the exchange of parts of the binary code sets to generate a new portfolio's variant.

Mutation is a random change of one or more positions of gens in the chromosome.

Fitness function defined in (11) is constructed to determine the optimal chromosome. It takes into account the specificity of the problem: getting the maximum profit on the investment portfolio, a desire of investors to invest full amount of money into a portfolio or take some balance to create a portfolio, as well as strong or not strong control of allocated funds. The latter implies the possibility of increasing the size of the initial investment to purchase an additional number of shares in perspective in terms of return on assets and/or securities.

Below the optimization algorithm is described.

B. *Optimization algorithm*

1) Generation of initial sets of investment portfolios. Generating of initial population with N chromosomes. Reading input data of investment companies, parameters of the genetic algorithm. Recoding of investment portfolio data into binary code. For example, each variant of an investment portfolio is encoded by a separate chromosome expressed by a binary code.

2) Checking of portfolios under optimality and their selection.

1 step. Calculation the value of the adaptability function for a given set of chromosomes (variants of realization of an investment portfolio), i.e. obtaining sets of solutions of the optimization problem.

2 step. The portfolios are selected in accordance with the threshold value of the function of adaptability. The goal of optimization - to get the maximum of investments income, i.e. achieving the maximum of the adaptability function .

3) Clustering (Grouping of Similar Investment Portfolios). Grouping the sets of solutions for the optimization that are encoded by chromosomes with similar phenotypes. The degree of proximity is the binary metric (Hamming).

4) Crossover. Generation of the new sets of investment portfolios, i.e. the selection the chromosomes and the creation a new set of subsidiary chromosomes. The result is a new set of investment portfolios, they are compared with the obtained already.

5) Mutation. Random change of genes in the chromosome for preventing falling into a local extreme and avoiding premature convergence.

6) Run steps (3-5) to generate a new generation of N chromosome population, that is, new variants of the investment portfolio.

7) Run steps (2-5) until the end condition of the algorithm (for example, a certain number of iterations) is reached.

IV. MODEL REALIZATION

The OptPortfolio software was developed for finding the optimal investment portfolio is based on the proposed algorithm. It was developed on the base of the crossplatform software development framework Qt 5.4 under the GNU LGPL license. Program product was realized with programming language C++. The software implements genetic operators of crossover, mutation, selection, parental selection, and clustering module.

In general this algorithmic tool needs the following input data: number and names of the companies; data of the securities of which an investor wants to buy; limitations on the number of securities (Minimum and maximum number); yield of securities of each company; cluster radius (size of the portfolios); number of iterations (populations) - the number of choices of portfolios; number of individuals in the population (number of variants in a single portfolio); probability of mutation (probability of random changes of the ammount of securities of i-the company where i=1…n, n - number of companies in the portfolio.)

Crossover produses two parental individuals, four subsidiaries are formed. Parental individuals - two randomly selected portfolios. We use multi-point crossbreeding namely in-gene. Number of shares of *i*-th securities (*i=1..n, n*-number of companies) in the "child's" portfolio with a probability of 50% may be inherited from so-called "father" portfolio, or with the same probability from "mother". It is important that when creating a new population, the centroids of clusters that are the "best" portfolios in each of the groups of similar portfolios, can not be interrupted, mutated or deleted. So the clusters are copied from the previous population to the next one. The new population consists of both parents and daughters.

During Selection process after reaching a predetermined number of generations, a subpopulation of clusters contains various solutions with different optimality. The group of the solutions defining the global extremum (at certain level of the solution of the problem) are centroids Z_{C_i} - the best portfolios of the last cluster’s subpopulation.

When Mutation is executed, it should be noted that the choice of values for a control parameter such as a cluster radius can only be carried out experimentally, taking into account the following recommendations: clusters should cover the whole search area; the value of the cluster radius should be sufficient to generate the clusters that cover all the optimal solutions; an intensive mutation should be realized with a large number of generations.

The obtained result includes data sets of the number of securities of each company, which are the solutions of the multi-extremity optimization problem of investment portfolio; amount of funds invested into the securities, i.e. the total price of the portfolio; the objective fitness function's value for each of these solutions, i.e. predictable profit. One of the program realization is presented in Table I in order to obtain for the optimal plan. The first stage is data preparation and preprocessing, the real data of prices and profitability of securities of several Ukrainian companies engaged in various fields were taken from the information portal of investment and finance.

TABLE I. INPUT DATA

Number of the company	Min amount of assets	Max amount of assets	Asset's price	Returns
1	10	20	266	4,3
2	100	500	0,15	0,04
3	200	700	0,8	0,01
4	20	200	22	10
5	50	300	8	4

The following parameters of the genetic algorithm were entered: the radius of the cluster 10; number of iterations 5000; probability of mutations 0.2; number of individuals in the population 50, initial capital 5000.

TABLE II. OPTIMAL PLAN

1	2	3	4	5	Portfolio's price	Return	Fitness function
12	136	202	55	52	5000	817	5,49

The resulting optimal plan is given on the Table II.

V. CONCLUSION

In this paper, a dynamic state-space model of investment portfolio is developed. The basis of the constructed model is the method of dynamic analogies, which consists of applying the laws of dynamics to the chosen measure of motion. On the base of systems of ordinary differential equations, we propose a model for forecasting risk-free assets and an optimal capital market, which is used to solve the problem of investment distribution. The optimization algorithm for finding the maximum return on investment through the achievement of the maximum of the target function of adaptability is presented. The developed models have been tested on real data, and the problem of maximizing the investor's income is solved. In further research, the model for forecasting risk-free assets and optimal capital market will expand, risk assets will be considered, it is expedient to consider the task of identifying the parameters with the help of real statistical data. Also, the software will be improved, its capabilities will be complemented by a solution to the problem of minimizing risk.

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